Wrong-Way Risk in Funding Valuation Adjustments

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Outline

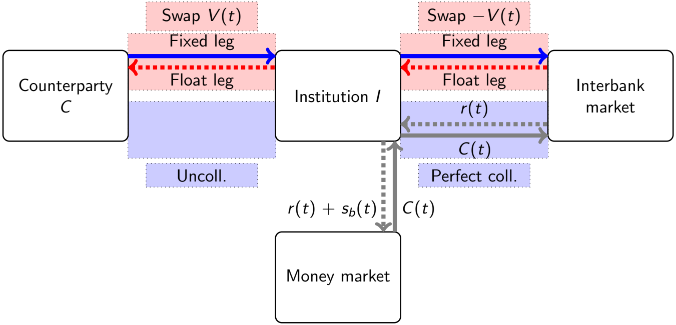
Goal: efficient Wrong-Way Risk (WWR) calculation for FVA.

Steps:

1. Introduce FVA and WWR;
2. Our contribution;
3. FVA equation;
4. Approximating FVA WWR;
5. Numerical results;
6. Conclusions.

FVA

- Suppose corporate C has a floating rate loan from bank I.
- To hedge IR risk, C often purchases an uncollateralized IR swap from I.
- I hedges in the interbank market, with perfect collateralization.
- I needs to fund itself in the money market at the cost of a funding spread $s_b(t)$ over $r(t)$.



FVA

- Valuation Adjustments (xVAs): CVA, DVA, FVA, MVA, KVA. Economic value = risk-neutral value − xVA.
- FVA is the funding cost of eliminating market risk on non-perfectly collateralized deals.
- FVA can be split into FBA and FCA.

FVA WWR

- WWR occurs when *"exposure to a counterparty is adversely correlated with the credit quality of that counterparty"* [1].
- FVA WWR: increase in funding risk as a consequence of increased market risk.
- Adverse relationship between IR and funding spreads.
- In our previous example of receiver swaps:
  - IR goes down.
  - Exposure goes up.
  - FVA goes up.
  - More negative funding spread sensitivity.
  - In addition, funding spreads will go up due to adverse relationship between IR and funding spreads.
- This happened during the March 2020 financial distress.
- FVA WWR difficult to hedge.

Our contribution

- Demonstrate relevance of WWR in FVA modelling.
- Understand how various modelling choices affect FVA WWR.
- Propose efficient approximation of FVA WWR.
- Avoid simulating extra (correlated) dynamics for credit and funding spreads.

FVA equation

- Asymmetric funding spreads: $s_b(t) > 0$ and $s_l(t) = 0$, so $\mathrm{FBA}(t) = 0$, hence $\mathrm{FVA}(t) = \mathrm{FCA}(t)$.
- Choose correlated SDEs for processes $r(t)$, $\lambda_I(t)$ and $\lambda_C(t)$ $\Rightarrow \rho_{r,I}$ & $\rho_{r,C}$ & $\rho_{I,C}$.

$$\begin{aligned}
\mathrm{FVA}(t) &= \mathbb{E}\left[\int_{u=t}^{T\wedge\tau_I\wedge\tau_C} \mathrm{e}^{-\int_t^u r(v)\mathrm{d}v} s_b(u)\,(V(u))^+\,\mathrm{d}u \,\middle|\, \mathcal{G}(t)\right] \\
&= \int_{u=t}^{T} \mathbb{E}\left[\mathrm{e}^{-\int_t^u \lambda_I(v)+\lambda_C(v)\mathrm{d}v} \mathrm{e}^{-\int_t^u r(v)\mathrm{d}v} s_b(u)\,(V(u))^+ \,\middle|\, \mathcal{F}(t)\right]\mathrm{d}u \\
&=: \int_{u=t}^{T} \mathrm{EPE}(t;u)\mathrm{d}u,
\end{aligned}$$

where we assumed conditional independence of defaults ($\rho_{I,C} = 0$) and no defaults before t.

FVA equation - exposure

Split $\mathsf{EPE}(t;u)$ as follows:

$$\mathsf{EPE}(t;u) = \mathsf{EPE}^{\perp}(t;u) + \mathsf{EPE}^{\mathsf{WWR}}(t;u).$$

- Independent exposure $\mathsf{EPE}^{\perp}(t;u)$ is taken from existing xVA engine where WWR is absent.
- WWR exposure $\mathsf{EPE}^{\mathsf{WWR}}(t;u)$ is the quantity we approximate.

Similar split for FVA:

$$\begin{aligned}\mathsf{FVA}(t) &= \int_t^T \mathsf{EPE}^{\perp}(t;u)\mathrm{d}u + \int_t^T \mathsf{EPE}^{\mathsf{WWR}}(t;u)\mathrm{d}u \\ &=: \mathsf{FVA}^{\perp}(t) + \mathsf{FVA}^{\mathsf{WWR}}(t).\end{aligned}$$

FVA equation - credit adjustment effect

Including τ_I and/or τ_C in the FVA definition results in a credit adjustment effect:

$$\begin{aligned}\mathrm{FVA}(t) &= \mathbb{E}\left[\int_{u=t}^{T\wedge\tau_I\wedge\tau_C} \mathrm{e}^{-\int_t^u r(v)\mathrm{d}v} s_b(u)\,(V(u))^+\,\mathrm{d}u \,\middle|\, \mathcal{G}(t)\right]\\ &= \int_{u=t}^{T} \mathbb{E}\left[\mathrm{e}^{-\int_t^u \lambda_I(v)+\lambda_C(v)\mathrm{d}v}\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v} s_b(u)\,(V(u))^+ \,\middle|\, \mathcal{F}(t)\right]\mathrm{d}u.\end{aligned}$$

In case all quantities are independent:

$$\begin{aligned}&\mathrm{EPE}(t;u)\\ &= \mathrm{EPE}^{\perp}(t;u)\\ &= \mathbb{E}_t\left[\mathrm{e}^{-\int_t^u \lambda_I(v)+\lambda_C(v)\mathrm{d}v}\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v} s_b(u)\,(V(u))^+\right]\\ &= \mathbb{E}_t\left[\mathrm{e}^{-\int_t^u \lambda_I(v)\mathrm{d}v}\right]\cdot\mathbb{E}_t\left[\mathrm{e}^{-\int_t^u \lambda_C(v)\mathrm{d}v}\right]\cdot\mathbb{E}_t\left[s_b(u)\right]\cdot\mathbb{E}_t\left[\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v}\,(V(u))^+\right]\\ &= P_I(t,u)\cdot P_C(t,u)\cdot\mathbb{E}_t\left[s_b(u)\right]\cdot\mathbb{E}\left[\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v}\,(V(u))^+ \,\middle|\, \mathcal{F}(t)\right].\end{aligned}$$

FVA equation - credit adjustment effect

The $\text{FVA}^{\perp}(t)$ reduction can be substantial, illustrated by a 74 basis point reduction in this example, which is approximately a 70% decrease:

	τ_I excl.	τ_I incl.
τ_C excl.	107.64	95.31
τ_C incl.	36.10	33.63

Table: $\text{FVA}^{\perp}(t)$ for the various choices of including/excluding τ_I and/or τ_C.

FVA equation - relevance of WWR

The WWR/RWR effects are non-negligible, as ratio $\frac{\text{FVA}(t)}{\text{FVA}^{\perp}(t)}$ is significantly different from 1 for non-zero correlations.

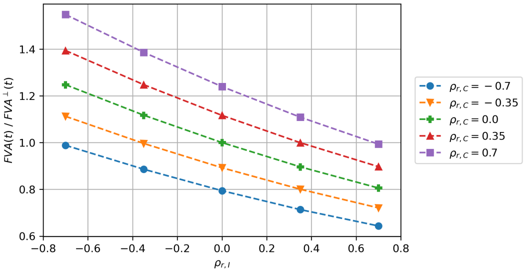


Figure: Correlation parameters effect for a receiver swap.

Model details - SDEs

Dynamics fit in the following generic setup:

$$
\begin{aligned}
\bar{z}(u) &= x_z(u) + b_z(u), \\
x_z(u) &= \mu_z(t,u) + y_z(t,u), \\
\int_t^u x_z(v)\mathrm{d}v &= M_z(t,u) + Y_z(t,u),
\end{aligned}
$$

where

- $\bar{z} \in \{r,\ \lambda_I,\ \lambda_C\}$ and $z \in \{r,\ I,\ C\}$.
- $b_z(u)$, $\mu_z(t,u)$ and $M_z(t,u)$ are deterministic quantities.
- $y_z(t,u)$ and $Y_z(t,u)$ are stochastic processes, with $\mathbb{E}_t\left[y_z(t,u)\right] = \mathbb{E}_t\left[Y_z(t,u)\right] = 0$.

Model details - funding spread

Credit-based funding spread, taking into account $\lambda_I(u)$ and liquidity adjustment $\ell(u)$:

$$\begin{aligned} s_b(u) &= \mathrm{LGD}_I\, \lambda_I(u) + \ell(u) \\ &= \mathrm{LGD}_I\, [x_I(u) + b_I(u)] + \ell(u) \\ &= \mathrm{LGD}_I\, [\mu_I(t,u) + b_I(u)] + \ell(u) + \mathrm{LGD}_I\, y_I(t,u) \\ &=: \mu_S(t,u) + \mathrm{LGD}_I\, y_I(t,u). \end{aligned}$$

WWR is introduced through the stochastic borrowing spread $s_b(u)$.

Model details - exposures

Now

$$
\begin{aligned}
&\mathrm{EPE}^{\perp}(t;u)\\
&\quad = P_I(t,u)P_C(t,u)\mu_S(t,u)\mathbb{E}_t\left[\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v}\left(V(u)\right)^+\right]\\
&\quad\quad + \mathrm{LGD}_I\,\mathbb{E}_t\left[\mathrm{e}^{-\int_t^u \lambda_I(v)+\lambda_C(v)\mathrm{d}v} y_I(t,u)\right]\mathbb{E}_t\left[\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v}\left(V(u)\right)^+\right],
\end{aligned}
$$

and

$$
\begin{aligned}
&\mathrm{EPE}^{\mathrm{WWR}}(t;u)\\
&= \mathbb{E}_t\left[\left(\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v}\left(V(u)\right)^+ - \mathbb{E}_t\left[\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v}\left(V(u)\right)^+\right]\right)\mathrm{e}^{-\int_t^u \lambda_I(v)+\lambda_C(v)\mathrm{d}v} s_b(u)\right].
\end{aligned}
$$

Model details - additional notation

- Some notation, with $\overline{z} \in \{r,\ \lambda_I,\ \lambda_C\}$ and $z \in \{r,\ I,\ C\}$:

$$\mathrm{e}^{-\int_t^u \overline{z}(v)\mathrm{d}v} = H_z(t,u)\mathrm{e}^{-Y_z(t,u)}$$
$$H_{z_1,\dots,z_n}(t,u) := H_{z_1}(t,u)\cdots H_{z_n}(t,u).$$

- Denote Taylor series expansions of e^{-x} as

$$T(x) := \sum_{j=0}^{\infty} \frac{(-x)^j}{j!},\ \ T_n^m(x) := \sum_{j=n}^{m} \frac{(-x)^j}{j!},$$

such that we can write $T(x) = T_0^n(x) + T_{n+1}^{\infty}(x)$.

Model details - FVA WWR exposure

Now apply the Taylor expansions:

$$\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v} = H_r(t,u)T(Y_r(t,u)),$$

$$\mathrm{e}^{-\int_t^u \lambda_I(v)+\lambda_C(v)\mathrm{d}v} = H_{I,C}(t,u)\left[T_0^1(Y_I(t,u)+Y_C(t,u)) + T_2^\infty(Y_I(t,u)+Y_C(t,u))\right].$$

Using the Taylor expansions and our model assumptions:

$$\begin{aligned}
&\mathrm{EPE}^{\mathrm{WWR}}(t;u)\\
&\quad = H_{r,I,C}(t,u)\mu_S(t,u)\mathbb{E}_t\left[T_0^{n_j}(Y_r(t,u))(-Y_I(t,u)-Y_C(t,u))\,(V(u))^+\right]\\
&\qquad + \mathrm{LGD}_I\, H_{r,I,C}(t,u)\mathbb{E}_t\left[T_0^{n_j}(Y_r(t,u))y_I(t,u)(1-Y_I(t,u)-Y_C(t,u))\,(V(u))^+\right]\\
&\qquad + \mathrm{LGD}_I\, H_{I,C}(t,u)\mathbb{E}_t\left[Y_I(t,u)y_I(t,u)\right]\mathbb{E}_t\left[\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v}\,(V(u))^+\right]\\
&\qquad + \varepsilon^{\mathrm{WWR},1},
\end{aligned}$$

where $\varepsilon^{\mathrm{WWR},1}$ contains scaled truncation errors.

Model details - idea of approximation

- W.l.o.g. take $y_r(t,u)$ normally distributed (HW1F).
- IR swap payoff $V(u)$ can be written in terms of $y_r(t,u)$.
- Through a Jamshidian-like argument, $(V(u))^+$ is also expressed in terms of $y_r(t,u)$.
- Approximate $y_z(t,u)$ and $Y_z(t,u)$, $z \in \{r,\ I,\ C\}$, in terms of $y_r(t,u)$:

$$
\begin{aligned}
Y_z(t,u) &\approx \rho_{rz}\sqrt{\frac{\mathbb{V}\mathrm{ar}_t\left(Y_z(t,u)\right)}{\mathbb{V}\mathrm{ar}_t\left(y_r(t,u)\right)}}\, y_r(t,u) \\
&=: \rho_{r,z}\Sigma_t(Y_z(t,u))y_r(t,u)
\end{aligned}
$$

Model details - WWR exposure approximation

$$\gamma(t,u) := \rho_{r,I}\Sigma(y_I(t,u)),\ \alpha(t,u) := -\left[\rho_{r,I}\Sigma(Y_I(t,u)) + \rho_{r,C}\Sigma(Y_C(t,u))\right],$$
$$\nu(t,u) := \gamma(t,u)\alpha(t,u),\ \beta_j(t,u) := \frac{(-\Sigma(Y_r(t,u)))^j}{j!},$$

Approximate $\mathrm{EPE}^{\mathrm{WWR}}(t;u)$ as follows.

$$\begin{aligned}
&\mathrm{EPE}^{\mathrm{WWR}}(t;u) \\
&= H_{r,I,C}(t,u)\left(\mu_S(t,u)\alpha(t,u) + \mathrm{LGD}_I\,\gamma(t,u)\right)\sum_{j=0}^{n_j}\beta_j(t,u)\mathbb{E}_t\left[y_r^{j+1}(t,u)\left(V(u)\right)^+\right] \\
&\quad + \mathrm{LGD}_I\,H_{r,I,C}(t,u)\nu(t,u)\sum_{j=0}^{n_j}\beta_j(t,u)\mathbb{E}_t\left[y_r^{j+2}(t,u)\left(V(u)\right)^+\right] \\
&\quad + \mathrm{LGD}_I\,H_{I,C}(t,u)\mathbb{E}_t\left[Y_I(t,u)y_I(t,u)\right]\mathbb{E}_t\left[\mathrm{e}^{-\int_t^u r(v)\mathrm{d}v}\left(V(u)\right)^+\right] + \varepsilon^{\mathrm{WWR},2}.
\end{aligned}$$

where $\varepsilon^{\mathrm{WWR},2} := \varepsilon^{\mathrm{WWR},1} + \varepsilon_{\mathrm{IV}}$ s.t. equality holds.

Recognize WWR and RWR.

Model details - WWR exposure approximation

- Until now, no assumptions have been made about product V. Write $\mathbb{E}_t\left[y_r^I(t,u)\,(V(u))^+\right]$ as a function of $y_r(t,u)$:

$$\mathbb{E}_t\left[y_r^I(t,u)\,(V(u))^+\right] = f\left(y_r(t,u)\right) + \varepsilon_{\mathrm{V}}.$$

  Product-level truncation error ε_{V} manifests itself after the application of the Gaussian approximation.
- For an IR swap under the HW1F model, $\mathbb{E}_t\left[y_r^I(t,u)\,(V(u))^+\right]$ can be written in terms of moments of a normal and truncated normal random variable.

Model details - approximation error

The approximation is a direct result of omitting overall error $\varepsilon^{\text{WWR},3} := \varepsilon^{\text{WWR},1} + \varepsilon_{\text{IV}} + \varepsilon_{\text{V}}$, where:

- $\varepsilon^{\text{WWR},1}$ is a truncation error;
- ε_{IV} is the Gaussian approximation error;
- ε_{V} is the product-level truncation error.

Numerical results - exposure profile

Example for IR swap under HW1F for IR and CIR++ for credit processes.

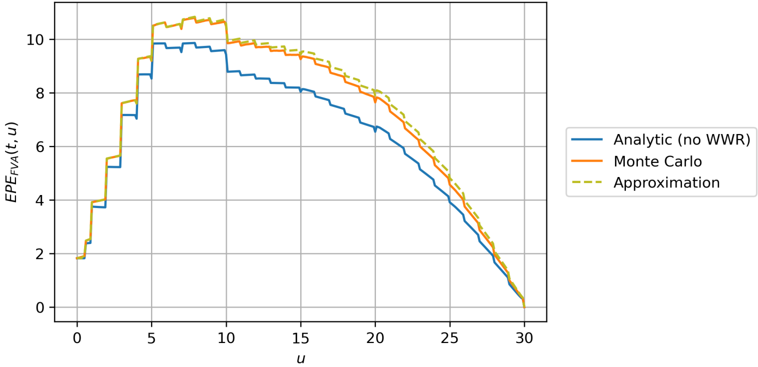


Figure: ITM receiver swap, $N = 10000$, EUR overnight yield curve, high credit rating for I, low credit rating for C, τ_I excluded, τ_C excluded.

Numerical results - WWR exposure

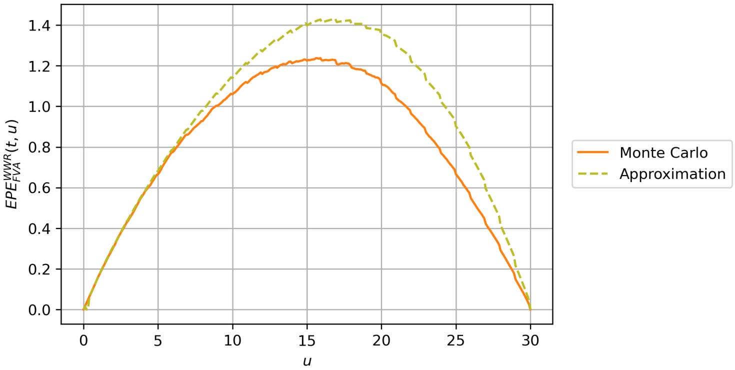


Figure: ITM receiver swap, $N = 10000$, EUR overnight yield curve, high credit rating for I, low credit rating for C, τ_I excluded, τ_C excluded.

Numerical results - FVA numbers

	FVA(t)	FVA$^{\text{WWR}}$(t)	WWR %	WWR runtime (sec)
Analytic (no WWR)	193.3481	0.0000	0.0000	0.00
Monte Carlo	217.8058	24.4577	12.6496	5.97
Approximation	221.6997	28.3516	14.6635	0.25

Table: ITM receiver swap, $N = 10000$, EUR overnight yield curve, high credit rating for I, low credit rating for C, τ_I excluded, τ_C excluded.

Conclusions

Conclusions

1. We demonstrated the relevance of WWR in FVA calculations.
2. We understand impact of various modelling choices.
3. We propose an efficient approximation:
   a. The approximation does not affect the no-WWR valuation.
   b. Build on top of existing xVA infrastructure, no extra simulation.
   c. Efficient method.
4. Example for IR swap under HW1F for IR and CIR++ for credit processes.
5. Extendable to other products and asset classes.

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