Valuation Adjustments and the hedging thereof in practice

T. van der Zwaard,[@*†] L.A. Grzelak,[@*‡] C.W. Oosterlee[@*§]

[@]T.vanderZwaard@tudelft.nl, [*]Delft Institute of Applied Mathematics, Delft University of Technology, [†]Pricing Model Validation, Rabobank,
[‡]Financial Engineering, Rabobank, [§]CWI - National Research Institute for Mathematics and Computer Science

Research objective

Understand the impact of xVA's from a bank's point of view and make the intuition behind it more tangible.

Portfolio equations

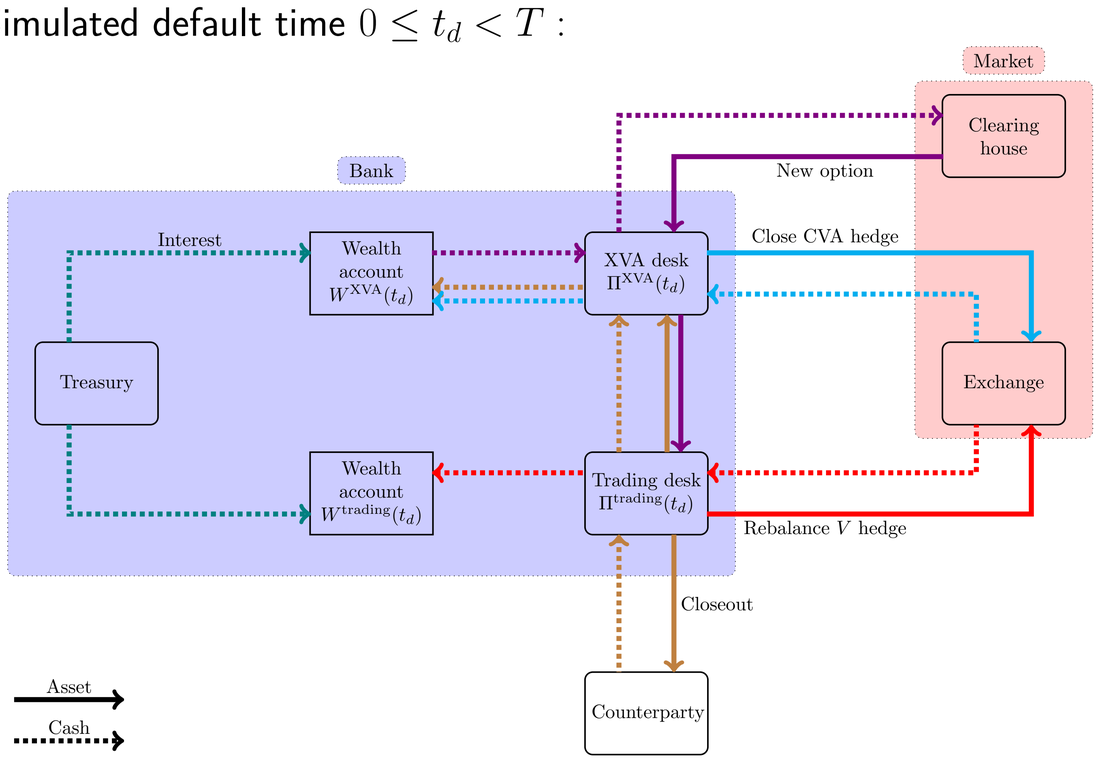
- Risky option $\hat{V}(t) = V(t) - \mathrm{CVA}(t)$.
- $V(t)$ is a risk-free European option on stock $S(t)$.
- $\mathrm{CVA}(t)$ with recovery rate R.
- Portfolio $\Pi(t)$ representing the strategy:

$$\begin{aligned}\Pi(t) &= V(t) - \mathrm{CVA}(t) - \left[\Delta(t) - \overline{\Delta}(t)\right] S(t). \\ &= V(t) - \mathrm{CVA}(t) - \left[\frac{\partial V(t)}{\partial S(t)} - \frac{\partial\, \mathrm{CVA}(t)}{\partial S(t)}\right] S(t).\end{aligned}$$

- Wealth account $W(t)$ represents the total wealth realized this strategy over time.

Cash flows at default

Simulated default time $0 \leq t_d < T$:



Merton jump-diffusion dynamics

$$\mathrm{d}\log S(t) = \left(r - \xi_J \mathbb{E}\left[e^J - 1\right] - \frac{1}{2}\sigma^2\right)\mathrm{d}t + \sigma \mathrm{d}B(t) + J\mathrm{d}X_J(t),$$

where $X_J(t)$ is a Poisson process with intensity ξ_J and jump magnitude J follows distribution $J \sim N\left(\mu_J, \sigma_J^2\right)$.

Output measures

At maturity T we examine the mean and variance of the following quantities:

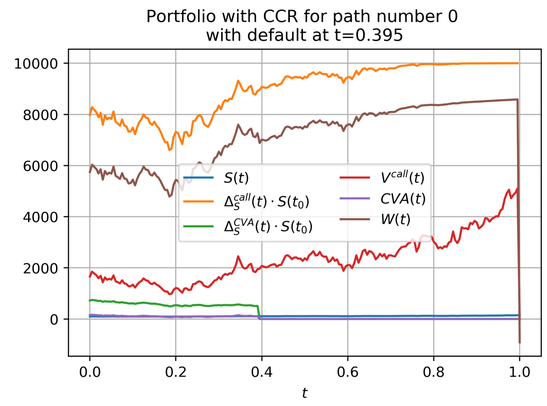
- $\Pi(T) + W(T)$
- $\mathrm{P\&L}\mathsf{Portfolio}(T)$
- $\mathrm{P\&L}\mathsf{Unexplained}(T)$

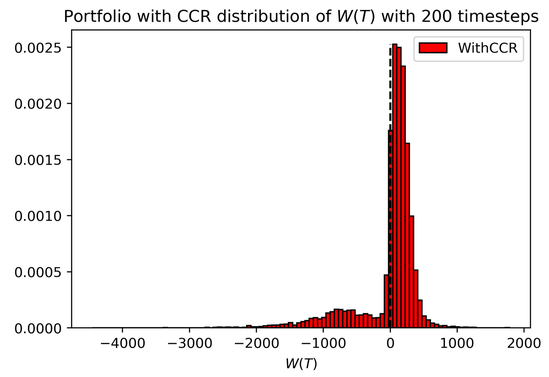
Numerical experiments

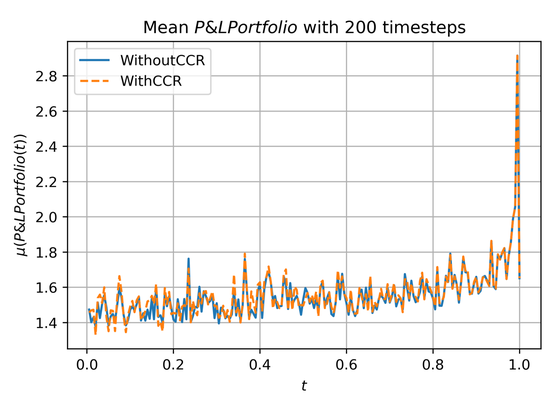
- Simulate market ($S(t)$ and $V(t)$) and calibrate the chosen model.
- Value the portfolio.
- Compare cases with and without Counterparty Credit Risk by means of simulated defaults (new CCR-free deal entered upon default).

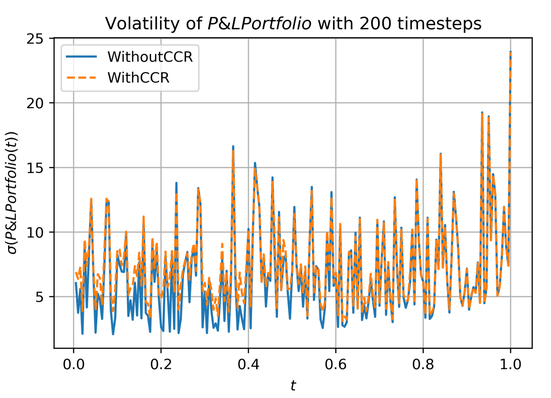
Market	Hedging instruments	Model
Black-Scholes (BS)	Stock	BS
Merton	Stock	BS
Merton	Stock	Merton
Merton	Stock, 1 option	Merton
Merton	Stock, 3 options	Merton

Results









Conclusions

- CVA is indeed a fair compensation for the CCR, otherwise a guaranteed loss.
- CVA market risk needs to be actively managed.
- In order to hedge the jump risk from the Merton model, the delta-hedge is insufficient.

Future work

Insights in well-known BS framework provide a clear foundation for future work:

- Incorporating additional features (collateral, managing credit risk of CVA, WWR).
- Portfolio of interest rate derivatives.
- Other valuation adjustments such as DVA, FVA and MVA.
- Impact of the bank's desk structure.